# Topic 14

# Exponentials and Logarithms

Bronze, Silver, Gold and
Platinum Worksheets
for AS Level Mathematics

## Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

## Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

## Quick Links

(Press Ctrl, as you click with your mouse to follow these links)


- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme


The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.


- Platinum Questions
- Platinum Mark Schemes


## Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

# Bronze Questions

**Calculators may not be used**

The total mark for this section is 25

**Q1**

Find the exact solution to the equation

$\ln x + \ln 3 = \ln 6$,

**(Total for Question 1 is 2 marks)**

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**Q2**

Sketch the graph of

$$y = 3^x, \quad x \in \square$$

showing the coordinates of any points at which the graph crosses the axes.

**(Total for Question 2 is 2 marks)**

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**Q3**

Find the value of $x$ for which

$$\log_3(x - 2) = -1.$$

**(Total for Question 2 is 2 marks)**

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**Q4**

Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

**(3)**

(b) $\ln(2y + 5) = 2 + \ln(4 - y)$

**(4)**

**(Total for Question 4 is 7 marks)**

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**Q5**

Given that

$$2\log_2 (x + 15) - \log_2 x = 6$$

(a) Show that

$x^2 - 34x + 225 = 0$

**(5)**

(b) Hence, or otherwise, solve the equation

$$2\log_2(x + 15) - \log_2 x = 6$$

**(2)**

**(Total for Question 5 is 7 marks)**

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**Q6**

Water is being heated in an electric kettle. The temperature, $\theta$ °C, of the water $t$ seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \quad 0 \leqslant t \leqslant T$$

(a) State the value of $\theta$ when $t = 0$

**(1)**

Given that the temperature of the water in the kettle is 70°C when $t = 40$,

(b) find the exact value of $\lambda$, giving your answer in the form $\dfrac{\ln a}{b}$, where $a$ and $b$ are integers.

**(4)**

**(Total for Question 6 is 5 marks)**

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**Bronze Mark Scheme**

**Q1.**

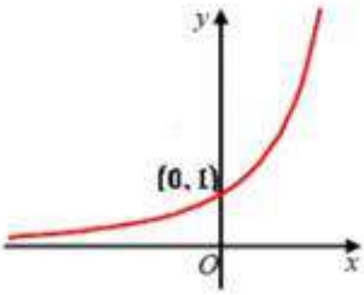
| Question Number | Scheme | Marks |
|---|---|---|
| | $\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$ | M1 |
| | $x = 2$ (only this answer) | A1 (cso) (2) |
| | | (2 marks) |

Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1

Note: $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

**Q2.**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | Graph of $y = 3^x$ and solving $3^{2x} - 9(3^x) + 18 = 0$ | | |
| |  | At least two of the three criteria correct. **(See notes below.)** | B1 |
| | | All three criteria correct. **(See notes below.)** | B1 |
| | | **Criteria number 1**: Correct shape of curve for $x \geq 0$ and at least touches the positive $y$-axis. **Criteria number 2**: Correct shape of curve for $x < 0$. Must not touch the x-axis or have any turning points. **Criteria number 3**: $(0, 1)$ stated or in a table or 1 marked on the $y$-axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the "correct" place on the $y$-axis. | |
| | | | [2] |
| | | | **Total 2** |

**Q3.**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $(x-2)=3^{-1}$ | $(x-2)=3^{-1}$ or $\frac{1}{3}$ | M1 oe |
| | $x\left\{=\frac{1}{3}+2\right\}=2\frac{1}{3}$ | $2\frac{1}{3}$ or $\frac{7}{3}$ or $2.\dot{3}$ or awrt 2.33 | A1 |
| | | | [2] |

M1: Is for correctly eliminating log out of the equation.

**Eg 1:** $\log_3(x-2)=\log_3(\frac{1}{3}) \Rightarrow x-2=\frac{1}{3}$ only gets M1 when the logs are correctly removed.

**Eg 2:** $\log_3(x-2)=-\log_3(3) \Rightarrow \log_3(x-2)+\log_3(3)=0 \Rightarrow \log_3(3(x-2))=0$

$\Rightarrow 3(x-2)=3^0$ only gets M1 when the logs are correctly removed,

but $3(x-2)=0$ would score M0.

**Note**: $\log_3(x-2)=-1 \Rightarrow \log_3\left(\frac{x}{2}\right)=-1$ $\frac{x}{\cdot}=3^{-1}$ would score M0 for incorrect use of logs.

**Alternative: changing base**

$\frac{\log_{10}(x-2)}{\log_{10}3}=-1 \Rightarrow \log_{10}(x-2)=-\log_{1}$ $\log_{10}(x-2)+\log_{10}3=0$

$\Rightarrow \log_{10}3(x-2)=0 \Rightarrow 3(x-2)=10^0$. point M1 is scored.

A correct answer in (b) without any workir es M1A1.

**Q4.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $e^{3x-9}=8\Rightarrow 3x-9=\ln 8$ | M1 |
| | $\Rightarrow x=\dfrac{\ln 8+9}{3}, =\ln 2+3$ | A1, A1 |
| | | (3) |
| (b) | $\ln(2y+5)=2+\ln(4-y)$ | |
| | $\ln\left(\dfrac{2y+5}{4-y}\right)=2$ | M1 |
| | $\left(\dfrac{2y+5}{4-y}\right)=e^2$ | M1 |
| | $2y+5=e^2(4-y)\Rightarrow 2y+e^2y=4e^2-5\Rightarrow y=\dfrac{4e^2-5}{2+e^2}$ | dM1, A1 |
| | | (4) |
| | | **7 marks** |

(a)

M1 Takes ln's of both sides and uses the power law. You may even accept candidates taking logs of both sides

A1 A correct unsimplified answer $\dfrac{\ln 8+9}{3}$ or equivalent such as $\dfrac{\ln 8e^9}{3}$, $3+\ln\left(\sqrt[3]{8}\right)$, $\dfrac{\log 8}{3\log e}+3$ or even 3.69

A1 cso $\ln 2+3$. Accept $\ln 2e^3$

Alt I (a)

$e^{3x-9}=8\Rightarrow\dfrac{e^{3x}}{e^9}=8\Rightarrow e^{3x}=8e^9\Rightarrow 3x=\ln\left(8e^9\right)$ for M1 (Condone slips on index work and lack of bracket)

Alt II (a)

$e^{x-3}=\sqrt[3]{8}\Rightarrow x-3=\ln\left(\sqrt[3]{8}\right)$ for M1 (Condone slips on the 9. Eg $e^{x-9}=2\Rightarrow x-9=\ln 2$)

(b)

M1 Uses a correct method to combine two terms to create a single ln term.

Eg. Score for $2+\ln(4-y)=\ln\left(e^2(4-y)\right)$ or $\ln(2y+5)-\ln(4-y)=\ln\left(\dfrac{2y+5}{4-y}\right)$

Condone slips on the signs and coefficients of the terms, but not on the $e^2$

M1 Scored for an attempt to undo the ln's to get an equation in $y$ This must be awarded after an attempt to combine the ln terms. Award for $\ln(g(y))=2\Rightarrow g(y)=e^2$ and can be scored eg where $g(y)=2y+5-(4-y)$

It cannot be awarded for just $2y+5=e^2+4-y$ where the candidate attempts to undo term by term

dM1 Dependent upon **both** previous M's. It is for making $y$ the subject. Expect to see both terms in $y$ collected and factorised (may be implied) before reaching $y=$. Condone slips, for eg, on signs. $y=2.615$ scores this.

A1 $y=\dfrac{4e^2-5}{2+e^2}$ or equivalent such as $y=4-\dfrac{13}{2+e^2}$ ISW after you see the correct answer.

Special Case: $\ln(2y+5)-\ln(4-y)=2\Rightarrow\dfrac{\ln(2y+5)}{\ln(4-y)}=2\Rightarrow\dfrac{2y+5}{4-y}=e^2\Rightarrow$ Correct answer score M0 M1 M1 A0

**Q5.**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | | | |
| **(a)** | $2\log(x+15)=\log(x+15)^2$ | | B1 |
| | $\log(x+15)^2-\log x=\log\dfrac{(x+15)^2}{x}$ | Correct use of $\log a-\log b=\log\dfrac{a}{b}$ | M1 |
| | | | |
| | | | |
| | $2^6=64$ or $\log_2 64=6$ | 64 used in the correct context | B1 |
| | $\log_2\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=64$ | Removes logs correctly | M1 |
| | | | |
| | | | |
| | $\Rightarrow x^2+30x+225=64x$ <br> *or* $x+30+225x^{-1}=64$ | Must see expansion of $(x+15)^2$ to score the final mark. | |
| | $\therefore x^2-34x+225=0$ * | | A1 |
| | | | **(5)** |
| **(b)** | $(x-25)(x-9)=0\Rightarrow x=25$ or $x=9$ | M1: Correct attempt to solve the given quadratic as far as $x=...$ <br> A1: Both 25 and 9 | M1 A1 |
| | | | **(2)** |
| | | | [7] |
| | | | |

**Q6.**

| Question Number | Scheme | Marks |
|---|---|---|
| **(a)** | $(\theta =)20$ | B1 **(1)** |
| **(b)** | Sub $t=40, \theta=70 \Rightarrow 70=120-100e^{-40\lambda}$ | |
| | $\Rightarrow e^{-40\lambda}=0.5$ | M1A1 |
| | $\Rightarrow \lambda=\dfrac{\ln 2}{40}$ | M1A1 |
| | | **(4)** |
| **Alt (b)** | Sub $t=40, \theta=70 \Rightarrow 100e^{-40\lambda}=50$ | |
| | $\Rightarrow \ln 100-40\lambda=\ln 50$ | M1A1 |
| | $\Rightarrow \lambda=\dfrac{\ln 100-\ln 50}{40}=\dfrac{\ln 2}{40}$ | M1A1 |
| | | **(4)** |

(a)

B1 Sight of $(\theta =)20$

(b)

M1 Sub $t=40, \theta=70 \Rightarrow 70=120-100e^{-40\lambda}$ and proceed to $e^{\pm 40\lambda}=A$ where $A$ is a constant. Allow sign slips and copying errors.

A1 $e^{-40\lambda}=0.5$ or $e^{40\lambda}=2$ or exact equivalent

M1 For undoing the e's by taking ln's and proceeding to $\lambda=..$

May be implied by the correct decimal answer awrt 0.017 or $\lambda=\dfrac{\ln 0.5}{-40}$

A1 cso $\lambda=\dfrac{\ln 2}{40}$

Accept equivalents in the form $\dfrac{\ln a}{b}$, $a,b\in\mathbb{Z}$ such as $\lambda=\dfrac{\ln 4}{80}$

# Silver Questions

**Calculators may not be used**

The total mark for this section is 28

**Q1**

Find the exact solutions, in their simplest form, to the equations

(a) $2 \ln (2x + 1) - 10 = 0$

**(2)**

(b) $3^x e^{4x} = e^7$

**(4)**

**(Total for Question 1 is 6 marks)**

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**Q2**

A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} + 2^4 - 9(2^x) = 0$$

$$\text{Let } \quad 2^x = y$$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \quad \text{or} \quad y = 1$$

$$\text{So } x = 3 \text{ or } x = 0$$

(a) Identify the two errors made by the student.

**(2)**

(b) Find the exact solution to the equation.

**(2)**

**(Total for Question 2 is 4 marks)**

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**Q3**

(a) Given that

$$2\log_3(x-5)-\log_3(2x-13)=1$$

show that $x^2 - 16x + 64 = 0$.

**(5)**

(b) Hence, or otherwise, solve $2\log_3(x-5)-\log_3(2x-13)=1$

**(2)**

**(Total for Question 1 is 7 marks)**

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**Q4**

Given that $a$ and $b$ are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

**(Total for Question 2 is 6 marks)**

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**Q5**

The mass, $m$ grams, of a leaf $t$ days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where $k$ and $p$ are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of $p$.

**(1)**

(b) Show that $k = \frac{1}{4}\ln 3$

**(4)**

**(Total for Question 5 is 5 marks)**

## Silver Mark Scheme

## Q1.

| Question Number | Scheme | Marks |
|---|---|---|
| **(a)** | $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \quad \Rightarrow 2x+1=e^5 \Rightarrow x=..$<br>$\Rightarrow x=\dfrac{e^5-1}{2}$ | M1<br>A1<br>(2) |
| **(b)** | $3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$<br>$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$<br>$x(\ln 3+4)=7 \Rightarrow x=...$<br>$x=\dfrac{7}{(\ln 3+4)}$ oe | M1,M1<br>dM1<br>A1<br>(4)<br>**6 marks** |
| **Alt 1 (b)** | $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$<br>$3^x=e^{7-4x} \Rightarrow x\ln 3=(7-4x)\ln e$<br>$x(\ln 3+4)=7 \Rightarrow x=...$<br>$x=\dfrac{7}{(\ln 3+4)}$ | M1,M1<br>dM1<br>A1<br>(4) |
| **Alt 2 (b) Using logs** | $3^x e^{4x}=e^7 \Rightarrow \log(3^x e^{4x})=\log e^7$<br>$\log 3^x+\log e^{4x}=\log e^7 \Rightarrow x\log 3+4x\log e=7\log e$<br>$x(\log 3+4\log e)=7\log e \Rightarrow x=...$<br>$x=\dfrac{7\log e}{(\log 3+4\log e)}$ | M1, M1<br>dM1<br>A1<br>(4) |
| **Alt 3 (b) Using** $\log_3$ | $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$<br>$3^x=e^{7-4x} \Rightarrow x=(7-4x)\log_3 e$<br>$x(1+4\log_3 e)=7\log_3 e \Rightarrow x=...$<br>$x=\dfrac{7\log_3 e}{(1+4\log_3 e)}$ | M1,M1<br>dM1<br>A1<br>(4) |
| **Alt 4 (b) Using** $3^x=e^{x\ln 3}$ | $3^x e^{4x}=e^7 \Rightarrow e^{x\ln 3}e^{4x}=e^7$<br>$\Rightarrow e^{x\ln 3+4x}=e^7, \Rightarrow x\ln 3+4x=7$<br>$x(\ln 3+4)=7 \Rightarrow x=... \qquad x=\dfrac{7}{(\ln 3+4)}$ | M1,M1<br>dM1 A1<br>(4) |

(a)

M1 Proceeds from $2\ln(2x+1)-10=0$ to $\ln(2x+1)=5$ before taking exp's to achieve $x$ in terms of $e^5$

Accept for M1 $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \Rightarrow x=f(e^5)$

Alternatively they could use the power law before taking exp's to achieve $x$ in terms of $\sqrt{e^{10}}$

$2\ln(2x+1)=10 \Rightarrow \ln(2x+1)^2=10 \Rightarrow (2x+1)^2=e^{10} \Rightarrow x=g\left(\sqrt{e^{10}}\right)$

A1 cso. Accept $x=\dfrac{e^5-1}{2}$ or other exact simplified alternatives such as $x=\dfrac{e^5}{2}-\dfrac{1}{2}$. Remember to isw.

The decimal answer of 73.7 will score M1A0 unless the exact answer has also been given.

The answer $\dfrac{\sqrt{e^{10}}-1}{2}$ does not score this mark unless simplified. $x=\dfrac{\pm e^5-1}{2}$ is M1A0

(b)

M1 Takes ln's or logs of both sides and applies the addition law.

$\ln(3^x e^{4x})=\ln 3^x+\ln e^{4x}$ or $\ln(3^x e^{4x})=\ln 3^x+4x$ is evidence for the addition law

If the $e^{4x}$ was 'moved' over to the right hand side score for either $e^{7-4x}$ or the subtraction law.

$\ln\dfrac{e^7}{e^{4x}}=\ln e^7-\ln e^{4x}$ or $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}} \Rightarrow 3^x=e^{7-4x}$ is evidence of the subtraction law

M1 Uses the power law of logs (seen at least once in a term with x as the index Eg $3^x, e^{4x}$ or $e^{7-4x}$ ).

$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ is an example after the addition law

$3^x=e^{7-4x} \Rightarrow x\log 3=(7-4x)\log e$ is an example after the subtraction law.

It is possible to score M0M1 by applying the power law after an incorrect addition/subtraction law

For example $3^x e^{4x}=e^7 \Rightarrow \ln(3^x)\times\ln(e^{4x})=\ln e^7 \Rightarrow x\ln 3\times 4x\ln e=7\ln e$

dM1 This is dependent upon **both** previous M's. Collects/factorises out term in $x$ and proceeds to $x=$.
Condone sign slips for this mark. An unsimplified answer can score this mark.

A1 If the candidate has taken ln's then they must use $\ln e=1$ and achieve $x=\dfrac{7}{(\ln 3+4)}$ or equivalent.

If the candidate has taken log's they must be writing log as oppose to ln and achieve

$x=\dfrac{7\log e}{(\log 3+4\log e)}$ or other exact equivalents such as $x=\dfrac{7\log e}{\log 3e^4}$.

**Q2.**

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| (a) | $2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$ | | B1 | 2.3 |
| | In line 4, $2^4$ has been replaced by 8 instead of by 16 | | B1 | 2.3 |
| | | | (2) | |
| (b) | Way 1<br>$2^{2x+4}-9(2^x)=0$<br>$2^{2x}\times 2^4-9(2^x)=0$<br>Let $2^x=y$<br>$16y^2-9y=0$ | Way 2<br>$(2x+4)\log 2-\log 9-x\log 2=0$ | M1 | 2.1 |
| | $y=\frac{9}{16}$ or $y=0$<br>So $x=\log_2\left(\frac{9}{16}\right)$ or $\dfrac{\log\left(\frac{9}{16}\right)}{\log 2}$<br>o.e. with no second answer. | $x=\dfrac{\log 9}{\log 2}-4$ o.e. | A1 | 1.1b |
| | | | (2) | |
| | | | | **(4 marks)** |

**Notes**

(a) B1: Lists error in line 2 (as above)

B1 : Lists error in line 4 (as above)

(b) M1: Correct work with powers reaching this equation

A1 : Correct answer here – there are many exact equivalents

**Q3.**

| Question Number | Scheme | Marks |
|---|---|---|
| | (a) $2\log_3(x-5) = \log_3(x-5)^2$ | B1 |
| | $\log_3(x-5)^2 - \log_3(2x-13) = \log_3 \dfrac{(x-5)^2}{2x-13}$ | M1 |
| | $\log_3 3 = 1$ seen or used correctly | B1 |
| | $\log_3\left(\dfrac{P}{Q}\right) = 1 \Rightarrow P = 3Q$ $\left\{\dfrac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13)\right\}$ | M1 |
| | $x^2 - 16x + 64 = 0$ (*) | A1 cso |
| | | (5) |
| | (b) $(x-8)(x-8) = 0 \Rightarrow x = 8$ <u>Must</u> be seen in part (b). | M1 A1 |
| | Or: Substitute $x = 8$ into original equation and verify. | |
| | Having additional solution(s) such as $x = -8$ loses the A mark. | (2) |
| | $x = 8$ with no working scores both marks. | 7 |

(a) Marks may be awarded if equivalent work is seen in part (b).

1st M: $\log_3(x-5)^2 - \log_3(2x-13) = \dfrac{\log_3(x-5)^2}{\log_3(2x-13)}$ is M0

$2\log_3(x-5) - \log_3(2x-13) = 2\log\dfrac{x-5}{2x-13}$ is M0

2nd M: <u>After the first mistake above</u>, this mark is available only if there is 'recovery' to the required $\log_3\left(\dfrac{P}{Q}\right) = 1 \Rightarrow P = 3Q$. Even then the final mark (cso) is lost.

<u>'Cancelling logs'</u>, e.g. $\dfrac{\log_3(x-5)^2}{\log_3(2x-13)} = \dfrac{(x-5)^2}{2x-13}$ will also lose the 2nd M.

<u>A typical wrong solution</u>:

$$\log_3\frac{(x-5)^2}{2x-13} = 1 \Rightarrow \log_3\frac{(x-5)^2}{2x-13} = 3 \Rightarrow \frac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13)$$

(Wrong step here)

This, with no evidence elsewhere of $\log_3 3 = 1$, scores B1 M1 B0 M0 A0

However, $\log_3\dfrac{(x-5)^2}{2x-13} = 1 \Rightarrow \dfrac{(x-5)^2}{2x-13} = 3$ is correct and could lead to full marks.

(Here $\log_3 3 = 1$ is implied).

<u>No log methods shown</u>:

It is <u>not</u> acceptable to jump immediately to $\dfrac{(x-5)^2}{2x-13} = 3$. The only mark this scores is the 1st B1 (by generous implication).

(b) M1: Attempt to solve the <u>given</u> quadratic equation (usual rules), so the factors $(x-8)(x-8)$ with no solution is M0.

**Q4.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$ or Way 2: $\log_3(9x) = \log_3 3^{a+2}$ | M1 |
| | $= 2 + a$ | A1 (2) |
| (b) | Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$ or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$ | M1 |
| | $\log x^5 = 5\log x$ or log81=4log3 or log81=4 ; $= \log_3 3^{5a-4}$ | M1 |
| | $= 5a - 4$ | A1 cso (3) |
| (c) | $\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$ | |
| | Method 1: $\Rightarrow 2 + a + 5a - 4 = 3$ ; Method 2: $\log_3\left(9x.\frac{x^5}{81}\right) =$ (3 or log27) | M1 |
| | $\Rightarrow a = \frac{5}{6}$ ; $\log_3\left(\frac{x^6}{9}\right) = 3$ or log 27 | A1 |
| | $\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a\log_{10} 3$ so $x =$ ; $\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$ | M1 |
| | $x = 2.498$ **or awrt** ; $x = 2.498$ **or awrt** | A1 (4) |
| | If $x$ = -2.498 appears as well or instead this is A0 | **Total 9** |
| | **Notes for Question** | |
| (a) | Way 1: M1: Use of log(ab) = log(a) + log(b) A1: must be $a + 2$ or $2 + a$<br>**Way 2:** Uses $x = 3^a$ to give $\log_3(9x) = \log_3 3^{a+2}$, A1 for $a + 2$ or $2 + a$ | |
| (b) | Way 1: M1: Use of log(a/b) = log(a) – log(b)<br>M1: Use of nlog(a) = log(a)$^n$<br>Way 2: M1 Use of correct powers of 3 in numerator and denominator<br>M1: Subtracts powers<br>A1: No errors seen | |
| (c) | **Method 1**: M1: Uses (a) and (b) results to form an equation in $a$ (may not be linear)<br>A1: $a$ = awrt 0.833<br>M1: Finds $x$ by use of 3 to a power, or change of base performed correctly<br>A1: $x = 2.498$ (accept answer which round to this value from 2.498049533…)<br>**Method 2:** M1: Use of log(ab) = log(a) + log(b) in an equation (RHS may be wrong)<br>A1: Equation correct and simplified<br>M1: Tries to undo log by 3 to power correctly, and uses root to obtain $x$<br>A1: $x = 2.498$ (accept answer which round to this value from 2.498049533…) Lose this mark if negative answer is given as well as or instead of positive answer. | |

| Question Number | Scheme | Marks |
|---|---|---|
| | Method 1 (Substituting a = 3b into second equation at some stage) | |
| | Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$ | M1 |
| | Substitution of $3b$ for $a$ (or a/3 for b) e.g. $\log_3 3b^2 = 2$ | M1 |
| | Using base correctly on **correctly derived** $\log_3 p = q$ e.g. $3b^2 = 3^2$ | M1 |
| | First correct value $b = \sqrt{3}$ (allow $3^{½}$) | A1 |
| | Correct method to find other value ( dep. on at least first M mark) | M1 |
| | Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$ | A1 |
| | Method 2 (Working with two equations in $\log_3 a$ and $\log_3 b$) | |
| | " Taking logs" of first equation and " separating" $\log_3 a = \log_3 3 + \log_3 b$ $(= 1 + \log_3 b)$ | M1 |
| | Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [ $\log_3 a = 1½$, $\log_3 b = ½$ ] | M1 |
| | Using base correctly to find a or b | M1 |
| | Correct value for $a$ or $b$ $a = 3\sqrt{3}$ or $b = \sqrt{3}$ | A1 |
| | Correct method for second answer, dep. on first M; correct second answer [Ignore negative values] | M1;A1 **[6]** |

| | | |
|---|---|---|
| Notes: | Answers must be exact; decimal answers lose both A marks<br>There are several variations on **Method 1**, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme.<br>In this method, the first three method marks on Epen are for<br>(i) First M1: correct use of log law,<br>(ii) Second M1: substitution of $a = 3b$,<br>(iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$<br><br>*Three examples of applying first 4 marks in Method 1:*<br>(i) $\log_3 3b + \log_3 b = 2$ gains second M1<br>$\log_3 3 + \log_3 b + \log_3 b = 2$ gains first M1<br>$(2\log_3 b = 1,\ \log_3 b = ½)$ no mark yet<br>$b = 3^{½}$ gains third M1, and if correct A1<br><br>(ii) $\log_3(ab) = 2$ gains first M1<br>$ab = 3^2$ gains third M1<br>$3b^2 = 3^2$ gains second M1<br>(iii) $\log_3 3b^2 = 2$ has gained first 2 M marks<br>$\Rightarrow 2\log_3 3b = 2$ or similar type of error<br>$\Rightarrow \log_3 3b = 1 \Rightarrow 3b = 3$ does not gain third M1, as $\log_3 3b = 1$ not derived correctly | |

Pearson

**Q5**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $p=7.5$ | B1 (1) |
| (b) | $2.5 = 7.5e^{-4k}$ | M1 |
| | $e^{-4k} = \frac{1}{3}$ | M1 |
| | $-4k = \ln(\frac{1}{3})$ | dM1 |
| | $-4k = -\ln(3)$ | |
| | $k = \frac{1}{4}\ln(3)$ | A1* |
| | See notes for additional correct solutions and the last A1 | (4) |
| | | 5 Marks |

# Gold Questions

**Calculators may not be used**

The total mark for this section is 30

**Q1**

(a) Find the positive value of $x$ such that

$$\log_x 64 = 2$$

**(2)**

(b) Solve for $x$

$$\log_2 (11 - 6x) = 2\log_2 (x - 1) + 3$$

**(6)**

**(Total for Question 1 is 8 marks)**

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**Q2**

Find algebraically the exact solutions to the equation $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\dfrac{a + \ln b}{c + \ln d}$ where $a$, $b$, $c$ and $d$ are integers.

**(Total for Question 2 is 5 marks)**

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**Q3**

(i)

$$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant}$$

Find $x$ in terms of $a$, giving your answer in its simplest form.

**(3)**

(ii)

$$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant}$$

Find $y$ in terms of $b$, giving your answer in its simplest form.

**(4)**

**(Total for Question 3 is 7 marks)**

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**Q4**

(a) Find the value of $y$ such that

$$\log_2 y = -3$$

**(2)**

(b) Find the values of $x$ such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

**(5)**

**(Total for Question 4 is 7 marks)**

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**Q5**

Rabbits were introduced onto an island. The number of rabbits, $P$, $t$ years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \ t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.

**(1)**

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

**(2)**

**(Total for Question 5 is 3 marks)**

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**Gold Mark Scheme**

**Q1.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $\log_x 64 = 2 \quad \Rightarrow \quad 64 = x^2$ | M1 |
| | So $x = 8$ | A1 (2) |
| (b) | $\log_2(11-6x) = \log_2(x-1)^2 + 3$ | M1 |
| | $\log_2\left[\dfrac{11-6x}{(x-1)^2}\right] = 3$ | M1 |
| | $\dfrac{11-6x}{(x-1)^2} = 2^3$ | M1 |
| | $\{11-6x = 8(x^2-2x+1)\}$ and so $0 = 8x^2 - 10x - 3$ | A1 |
| | $0 = (4x+1)(2x-3) \quad \Rightarrow x = \ldots$ | dM1 |
| | $x = \dfrac{3}{2}, \left[-\dfrac{1}{4}\right]$ | A1 (6) |
| | | [8] |
| (a) | M1 for getting out of logs<br>A1 Do not need to see $x = -8$ appear and get rejected. Ignore $x = -8$ as extra solution.<br>$x = 8$ with no working is M1 A1 | |
| (b) | 1[st] M1 for using the $n\log x$ rule<br>2[nd] M1 for using the $\log x - \log y$ rule or the $\log x + \log y$ rule as appropriate<br>3[rd] M1 for using 2 to the power– need to see $2^3$ or 8 (May see $3 = \log_2 8$ used)<br>**If all three M marks have been earned and logs are still present in equation do not give final M1.** So solution stopping at $\log_2\left[\dfrac{11-6x}{(x-1)^2}\right] = \log_2 8$ would earn M1M1M0<br>1[st] A1 for a correct 3TQ<br>4[th] dependent M1 for attempt to solve or factorize their 3TQ to obtain $x = \ldots$ (mark depends on three previous M marks)<br>2[nd] A1 for 1.5 (ignore -0.25)<br>s.c 1.5 only – no working – is 0 marks | |
| (a) | <u>Alternatives</u><br>Change base : (i) $\dfrac{\log_2 64}{\log_2 x} = 2$, so $\log_2 x = 3$ and $x = 2^3$, is M1 or<br>(ii) $\dfrac{\log_{10} 64}{\log_{10} x} = 2$, $\log x = \frac{1}{2}\log 64$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1<br>**BUT** $\log x = 0.903$ so $x = 8$ is M1A0 (loses accuracy mark)<br>(iii) $\log_{64} x = \frac{1}{2}$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1 | |

**Q2.**

| Question Number | Scheme | Marks |
|---|---|---|
| | Take $\log_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$ | M1 |
| | $x\ln 2 + (3x+1)\ln e = \ln 10$ | M1 |
| | $x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$ | dM1 |
| | and uses $\ln e = 1$ | M1 |
| | $x = \dfrac{-1+\ln 10}{3+\ln 2}$ | A1 |
| | | **(5)** |
| | **Note that the 4[th] M mark may occur on line 2** | |
| | | **( 5 marks)** |

**Notes for Question Continued**

(b)

M1 Takes logs of both sides **and** splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides **and** using the subtraction law.

M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1

$$\ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x\ln 2 \times (3x+1)\ln e = \ln 10$$

dM1 This is dependent upon both previous two M's being scored. It can be awarded for a full method to solve their linear equation in $x$. The terms in $x$ must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to $x = \ldots$

M1 Uses $\ln e = 1$. This could appear in line 2, but it must be part of their equation and not just a statement.

Another example where it could be awarded is $e^{3x+1} = \dfrac{10}{2^x} \Rightarrow 3x+1 = \ldots$

A1 Obtains answer $x = \dfrac{-1+\ln 10}{3+\ln 2} = \left(\dfrac{\ln 10 - 1}{3+\ln 2}\right) = \left(\dfrac{\log_e 10 - 1}{3+\log_e 2}\right)$ *oe*. **DO NOT ISW HERE**

Note 1: If the candidate takes $\log_{10}$'s of both sides can score M1M1dM1M0A0 for 3 out of 5.

$$\text{Answer} = x = \frac{-\log e + \log 10}{3\log e + \log 2} = \left(\frac{-\log e + 1}{3\log e + \log 2}\right)$$

Note 2: If the candidate writes $x = \dfrac{-1+\log 10}{3+\log 2}$ without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5.

| Question Number | Scheme | Marks |
|---|---|---|
| **Alt 1 to (b)** | Writes lhs in e's $\quad 2^x e^{3x+1} = 10 \Rightarrow e^{x\ln 2} e^{3x+1} = 10$ | 1st M1 |
| | $\Rightarrow e^{x\ln 2 + 3x+1} = 10, \quad x\ln 2 + 3x + 1 = \ln 10$ | 2nd M1, 4th M1 |
| | $x(\ln 2 + 3) = \ln 10 - 1 \Rightarrow x = ..$ | dM1 |
| | $x = \dfrac{-1 + \ln 10}{3 + \ln 2}$ | A1 **(5)** |

**Notes for Question Alt 1**

M1 Writes the lhs of the expression in e's. Seeing $2^x = e^{x\ln 2}$ in their equation is sufficient

M1 Uses the addition law on the lhs to produce a single exponential

dM1 Takes ln's of both sides to produce and attempt to solve a linear equation in $x$
You may condone slips in signs for this mark but the process must be correct leading to $x= ..$

M1 Uses $\ln e = 1$. This could appear in line 2

**Q3.**

| Question Number | Scheme | Marks |
|---|---|---|
| (i) | Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$ | M1 |
| | Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$<br>Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$ | M1 |
| | $(x=)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent) | A1cao<br>**(3)** |
| (ii) Way 1 | $\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ Applies quotient law of logarithms | M1 |
| | $\dfrac{(9y+b)}{(2y-b)} = 3^2$ Uses $\log_3 3^2 = 2$ | M1 |
| | $(9y+b) = 9(2y-b) \Rightarrow y =$ Multiplies across and makes $y$ the subject | M1 |
| | $y = \frac{10}{9}b$ | A1cso<br>**(4)** |
| Way 2 | Or : $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ 2nd M mark | M1 |
| | $\log_3(9y+b) = \log_3 9(2y-b)$ 1st M mark | M1 |
| | $(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ Multiplies across and makes $y$ the subject | M1<br>A1cso<br>**(4)**<br>**[7]** |

| | **Notes** |
|---|---|
| (i) | 1st M1: Applies power law of logarithms correctly to one side of the equation<br>M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x+a = \sqrt{16a^6}$ isw so allow $x+a = \pm\ 4a^3$ for Method mark. Also allow $x+a = 4a^4$ or $x+a = \pm\ 4a^{5.5}$ or even $x+a = 16a^3$ as there is evidence of attempted square root.<br>May see the correct $x+a = 10^{(\log 4 + 3\log a)}$ *so* $x = -a + 10^{(\log 4+3\log a)}$ which gains M1A0 unless followed by the answer in the scheme.<br>A1: Do not allow $x = \pm\ 4a^3 - a$ for accuracy mark. You may see the factorised $a(2a+1)(2a-1)$ o.e. |
| (ii) | M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in $y$<br>M1: Uses $\log_3 3^2 = 2$<br>3rd M1: Obtains **correct** linear equation in $y$ usually the one in the scheme and attempts $y =$<br>A1cso: $y = \frac{10}{9}b$ or correct equivalent after **completely correct** work.<br>**Special case:**<br>$\dfrac{\log_3(9y+b)}{\log_3(2y-b)} = 2$ is M0 unless clearly crossed out and replaced by the correct $\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$<br>Candidates may then write $\dfrac{(9y+b)}{(2y-b)} = 3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution. |

**Q4.**

| Question Number | Scheme | Marks | |
|---|---|---|---|
| Q (a) | $\log_2 y = -3 \quad \Rightarrow \quad y = 2^{-3}$ | M1 | |
| | $y = \frac{1}{8}$ or $0.125$ | A1 | (2) |
| (b) | $32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$ | M1 | |
| | [or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$] | | |
| | [or $\log_2 32 = \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \frac{\log_{10} 512}{\log_{10} 2}$] | | |
| | $\log_2 32 + \log_2 16 = 9$ | A1 | |
| | $(\log x)^2 = \ldots$ or $(\log x)(\log x) = \ldots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2) | M1 | |
| | $\log_2 x = 3 \quad \Rightarrow \quad x = 2^3 = 8$ | A1 | |
| | $\log_2 x = -3 \quad \Rightarrow \quad x = 2^{-3} = \frac{1}{8}$ | A1ft | (5) [7] |

| | |
|---|---|
| (a) | M1 for <u>getting out of logs</u> correctly.<br>If done by change of base, $\log_{10} y = -0.903\ldots$ is insufficient for the M1, but $y = 10^{-0.903}$ scores M1.<br>A1 for the <u>exact</u> answer, e.g. $\log_{10} y = -0.903 \Rightarrow y = 0.12502..$ scores M1 (implied) A0.<br><u>Correct answer</u> with no working scores both marks.<br><u>Allow</u> both marks for implicit statements such as $\log_2 0.125 = -3$. |
| (b) | 1st M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$, $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively).<br>1st A1 for 9 (exact).<br>2nd M1 for getting $(\log_2 x)^2$ = constant. The constant can be a log or a sum of logs.<br>If written as $\log_2 x^2$ instead of $(\log_2 x)^2$, allow the M mark <u>only</u> if subsequent work implies correct interpretation.<br>2nd A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0.<br>3rd A1ft for an answer of $\frac{1}{\text{their } 8}$. An ft answer may be non-exact.<br><u>Possible mistakes:</u><br>$\log_2(2^9) = \log_2(x^2) \Rightarrow x^2 = 2^9 \Rightarrow x = \ldots$ scores M1A1(implied by 9)M0A0A0<br>$\log_2 512 = \log_2 x \times \log_2 x \Rightarrow x^2 = 512 \Rightarrow x = \ldots$ scores M0A0(9 never seen)M1A0A0<br>$\log_2 48 = (\log_2 x)^2 \Rightarrow (\log_2 x)^2 = 5.585 \Rightarrow x = 5.145,\ x = 0.194$ scores M0A0M1A0A1ft<br><u>No working</u> (or 'trial and improvement'):<br>$x = 8$ scores M0 A0 M1 A1 A0 |

**Q5.**

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| Q | $P = 80e^{\frac{t}{5}}$ | | | |
| (a) | $t = 0 \Rightarrow P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$ | $\underline{80}$ | B1 | |
| | | | | (1) |
| (b) | $P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \dfrac{1000}{80} = e^{\frac{t}{5}}$ | Substitutes $P = 1000$ and rearranges equation to make $e^{\frac{t}{5}}$ the subject. | M1 | |
| | $\therefore\ t = 5\ln\left(\dfrac{1000}{80}\right)$ | | | |
| | | awrt $5\ln\left(\dfrac{1000}{80}\right)$ years | A1 | |
| | | | | (2) |
| | | | | [ 3 ] |

## Platinum Questions

**Calculators may not be used**

The total mark for this section is 25

**1** (a) Solve the equation

$$\sqrt{(3x + 16)} = 3 + \sqrt{(x + 1)}$$

**(5)**

(b) Solve the equation

$$\log_3 (x - 7) - \tfrac{1}{2} \log_3 x = 1 - \log_3 2$$

**(7)**

**(Total for Question 1 is 12 marks)**

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**2** (a) Given that $x > 0$, $y > 0$, $x \neq 1$ and $n > 0$, show that

$$\log_x y = \log_{x^n} y^n$$

**(2)**

(b) Solve the following, leaving your answers in the form $2^p$, where $p$ is a rational number.

(i) $\log_2 u + \log_4 u^2 + \log_8 u^3 + \log_{16} u^4 = 5$

(ii) $\log_2 v + \log_4 v + \log_8 v + \log_{16} v = 5$

(iii) $\log_4 w^2 + \dfrac{3\log_8 64}{\log_2 w} = 5$

**(9)**

**(Total for Question 2 is 11 marks)**

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**Platinum Mark Scheme**

| Q. | Scheme | Marks | Notes |
|---|---|---|---|
| 1(a) | $3x+16=9+x+1+6\sqrt{x+1}$ <br> $3+x=3\sqrt{x+1}$ (o.e.) <br><br> $9+6x+x^2=9(x+1)$ <u>or</u> $y=\sqrt{x+1}\to$ 3TQ in $y$ <br> $x^2-3x=0$ <u>or</u> $(y-2)(y-1)=0$ <br> <u>$x = 0$ or $3$</u> | M1 <br> A1 <br><br> M1 <br> A1 <br> B1 (5) | Initial squaring - both sides <br> Correct collecting of terms <br> 2nd squaring <br> o.e. |
| (b) | $\frac{1}{2}\log_3 x=\log_3\sqrt{x}$ <br> $\log_3(x-7)-\log_3\sqrt{x}=\log_3\frac{x-7}{\sqrt{x}}$ <br> So $2x-14=3\sqrt{x}$ (o.e. all $x$ terms on same line) <br> $2\left(\sqrt{x}\right)^2-3\sqrt{x}-14=0$ <br> $\left(2\sqrt{x}-7\right)\left(\sqrt{x}+2\right)=0$ <br> $\sqrt{x}=\frac{7}{2}$ or $-2$ <br> $\underline{x=\frac{49}{4}}$ | B1 <br> M1 <br> M1A1 <br> M1 <br> A1 <br> A1 (7) <br> **[12]** | Both values (S+ for checking values) <br> For use of $n\log x$ rule <br> For reducing $x$s to a single log <br> M1 for getting out of logs <br> A1 for correct equation <br> Attempt to solve suitable 3TQ in $x$ or $\sqrt{x}$ <br> Either solution for $\sqrt{x}$ or $x$. Must be rational $a/b$ <br> 49/4 oe only (S+ for clear reason for rejecting $x=4$) |

| Qu | Scheme | Mark |
|---|---|---|
| 2(a) | $\log_x y = k \Rightarrow x^k = y \Rightarrow y^n = \ldots$ <u>or</u> $\log_x y^n = nk \Rightarrow y^n = ..$ <u>or</u> base change | M1 |
| | $y^n = \left(x^k\right)^n = x^{nk} = \left(x^n\right)^k$ therefore $\log_{x^n} y^n = k = \log_x y$ **(*)** | A1cso |
| | | (2) |
| (b)(i) | LHS = $4\log_2 u$ | M1 |
| | $\therefore \log_2 u = \frac{5}{4}$ so $\underline{u = 2^{\frac{5}{4}}}$ | A1 |
| | | (2) |
| (ii) | $\log_{16} v^4 + \log_{16} v^2 + \log_{16} v^{\frac{4}{3}} + \log_{16} v$ <u>or</u> $\log_2 v + \log_2 v^{\frac{1}{2}} + \log_2 v^{\frac{1}{3}} + \log_2 v^{\frac{1}{4}}$ | M1 |
| | $= \log_{16} v^{\frac{25}{3}}$ $=$ $\log_2 v^{\frac{25}{12}}$ | M1 |
| | so $v = 2^{\frac{60}{25}} = \underline{2^{\frac{12}{5}}}$ | A1 |
| | | (3) |
| (iii) | LHS = $\log_2 w + \dfrac{3\times 2}{\log_2 w}$ | M1 |
| | Sub $t = \log_2 w$ gives $t^2 - 5t + 6 = 0$ or $(t-3)(t-2) = 0$ | M1 |
| | $\log_2 w = 2 \Rightarrow w = \underline{2^2}$ and $\log_2 w = 3 \Rightarrow w = \underline{2^3}$ (accept 4 and 8 ) | A1,A1 |
| | | (4) |
| | | **(11)** |